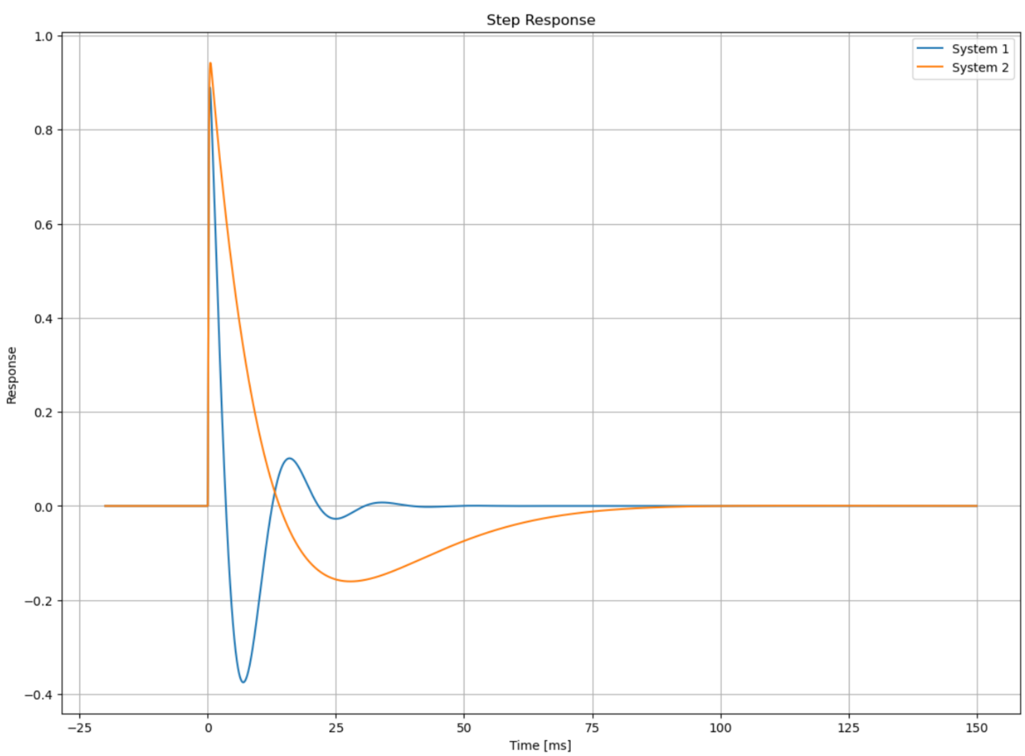
Step Response and Time Domain Behavior for Select Bass Alignments

By: Dan Cyr, January 2024

Introduction:

This paper explores very simplistic models of a handful of bass alignments modelled as various idealistic filter responses. These models are accurate up to a point – and should be considered small signal valid only. The motivation for this paper was repeatedly seeing incorrect interpretation of step response graphs as to time-domain behavior online in many places. In general, sealed boxes are modelled with a second order high-pass filter, while vented, passive radiator and transmission-line enclosures are modelled with fourth-order high-pass filters. For the fourth order systems I used Butterworth filters which have the maximum bass extension without passband ripple

As a teaser, here is the step response of 2 different systems. Which of the two systems would seem to have "faster" bass response to transients? Which would "decay" or "settle" faster? They have identical low-pass filters at 2000hz but are different in the high pass.



Introduction to Impulse and Step Responses of Filters

The impulse response of a filter is defined as its response to an input that is 0 for t<0 and t>0, but equal to infinity at t=0. An infinitely narrow pulse that is infinitely tall is a bit confusing, for the purpose of this paper it is acceptable to envision it as "very narrow and very tall".

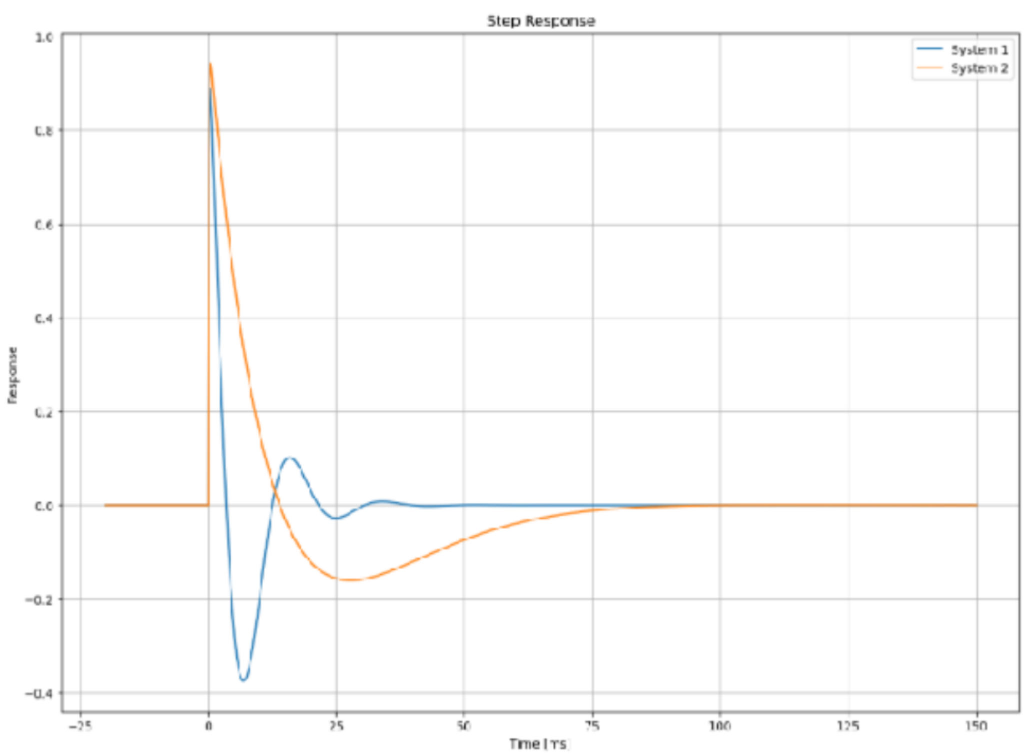
選擇性低音校準的階躍響應和時域行為

作者：**Dan Cyr** ，**2024** 年 **1** 月

簡介：

本文探討了少數低音校準的非常簡單模型，這些模型模擬為各種理想化的濾波器響應。這些模型準確到某個程度，並且應該僅被視為小訊號有效。撰寫本文的動機是反覆看到許多地方在網路上錯誤解讀階躍響應圖表，以了解時域行為。一般來說，密閉式音箱以二階高通濾波器建模，而通風式、被動式輻射器和傳輸線外殼則以四階高通濾波器建模。對於四階系統，我使用了具有最大低音延伸且無通帶漣波的巴特沃斯濾波器

作為預告，以下是 2 個不同系統的階躍響應。哪一個系統的瞬態低音響應看起來「較快」？哪一個會「衰減」或「穩定」得更快？它們在 2000hz具有相同的低通濾波器，但在高通濾波器方面有所不同。

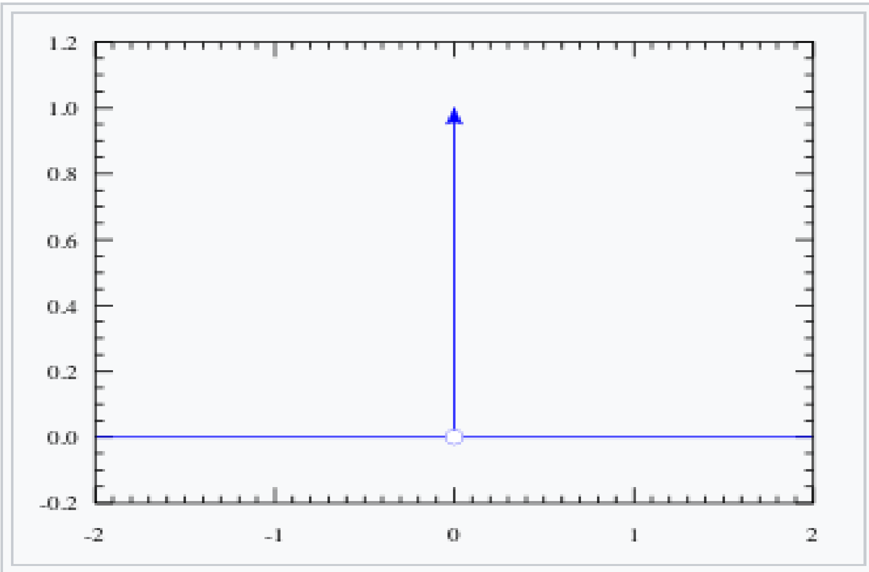


濾波器的脈衝和階躍響應簡介

濾波器的衝激響應定義為其對輸入的響應，輸入在 t<0 和 t>0 時為 0，但在 t=0 時等於無窮大。一個無限窄且無限高的脈衝有點令人困惑，為了本文的目的，可以將其視為「非常窄且非常高」。

acceptableto envisionit as “very narrowandvery tall”.

A graph of an ideal impulse sourced from Wikipedia is shown here:



For the calculus savvy, the integral of the ideal impulse (sometimes called the Dirac Delta function) is defined to have area=1. The integral of the impulse response also defines the step response.

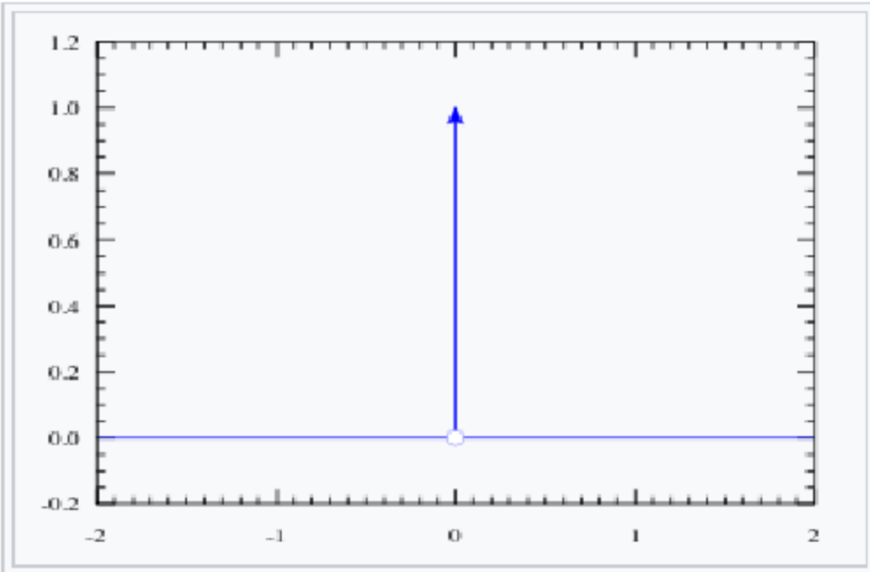
Step response is defined as response to input that is 0 for t<0 and 1 for t>0. The value at t=0 is treated as any of (0, 0.5, 1), for our case it does not matter. Sometimes called the unit step function or Heaviside step function.

An Aside:

Both the ideal impulse and unit step function are defined mathematically with calculus but also digitally approximated with a DAC. The impulse response in digital realm is all samples take value 0, and at time 0 a single sample of maximum digital value. In practice the impulse response is measured, and the integral performed to calculate the step function, rather than measuring the step response with a step input stimulus.

Here is a plot sourced from Wikipedia for the step function:

維基百科中顯示了一個理想脈衝圖表：



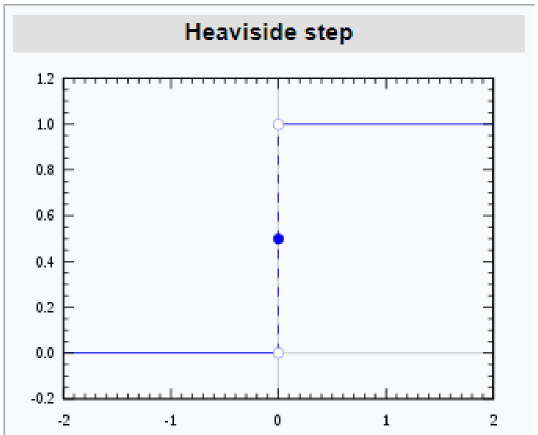
對於微積分專家來說，理想脈衝的積分（有時稱為狄拉克δ函數）定義為面積=1。脈衝響應的積分也定義了階躍響應。

階躍響應定義為對輸入的響應，輸入在t<0時為0，在t>0時為1。t=0時的值被視為(0, 0.5, 1)中的任何一個，在我們的案例中這並不重要。有時稱為單位階躍函數或赫維塞德階躍函數。

旁白：

理想脈衝和單位階躍函數在數學上以微積分定義，但也可以用 DAC 以數位方式近似。數位領域的脈衝響應是所有樣本取值為 0，而在時間 0 時單個樣本取最大數位值。在實務上，脈衝響應是測量的，並且執行積分以計算階躍函數，而不是用階躍輸入刺激來測量階躍響應。

以下是取自維基百科的階躍函數圖形：



The Heaviside step function, using the half-maximum convention

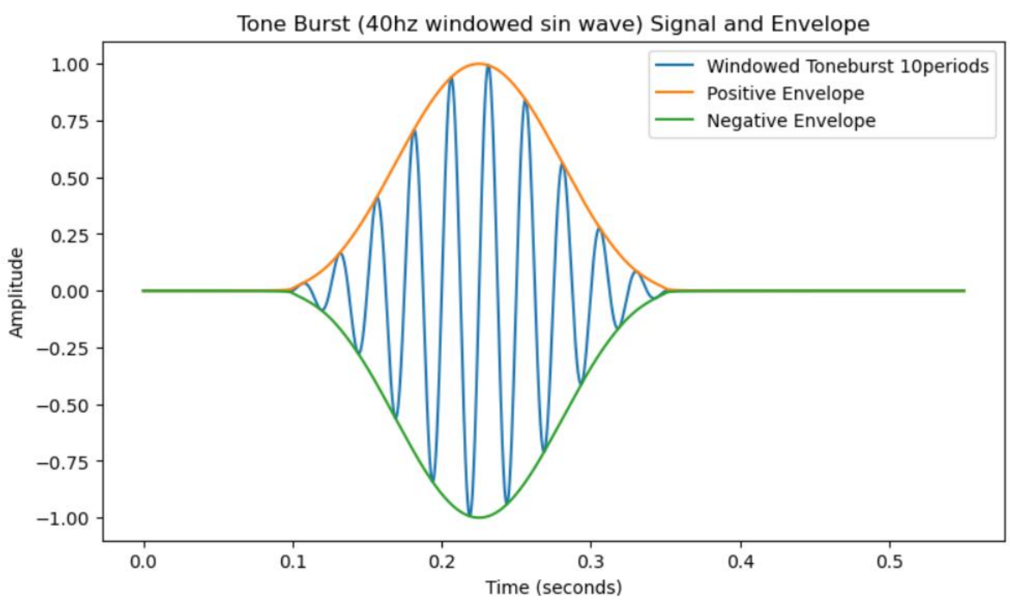
General information

General definition $$H(x) := \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

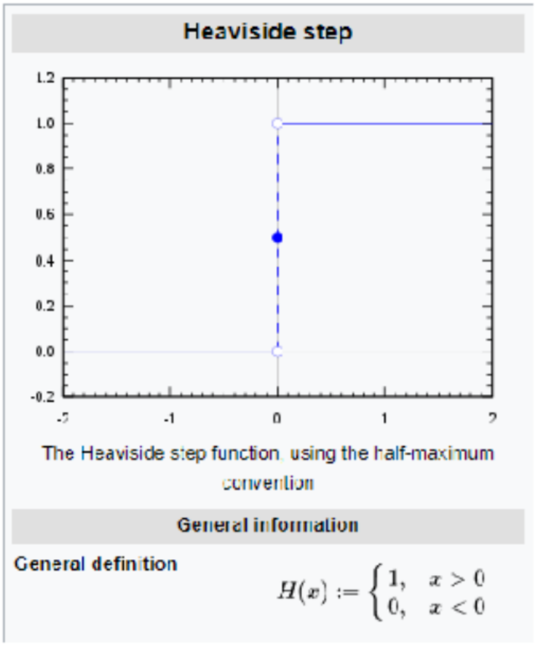
Some people would understandably pick "System 1" above as "settling" or "decaying" faster but in practice "System 2" has better time-domain performance. Intuitively, a system that lacks bass will have a step-response that settles to zero quicker, after all an infinite bandwidth system's step response is 1 for all t > 0. The time it takes step response to "return to zero" has more to do with its bass extension than time domain performance.

Introduction to the Envelope of Signal (nearly math free!)

I need to introduce the envelope of a signal which is a way of identifying the instantaneous amplitude of a rapidly changing complex signal. A picture is worth more than many words:



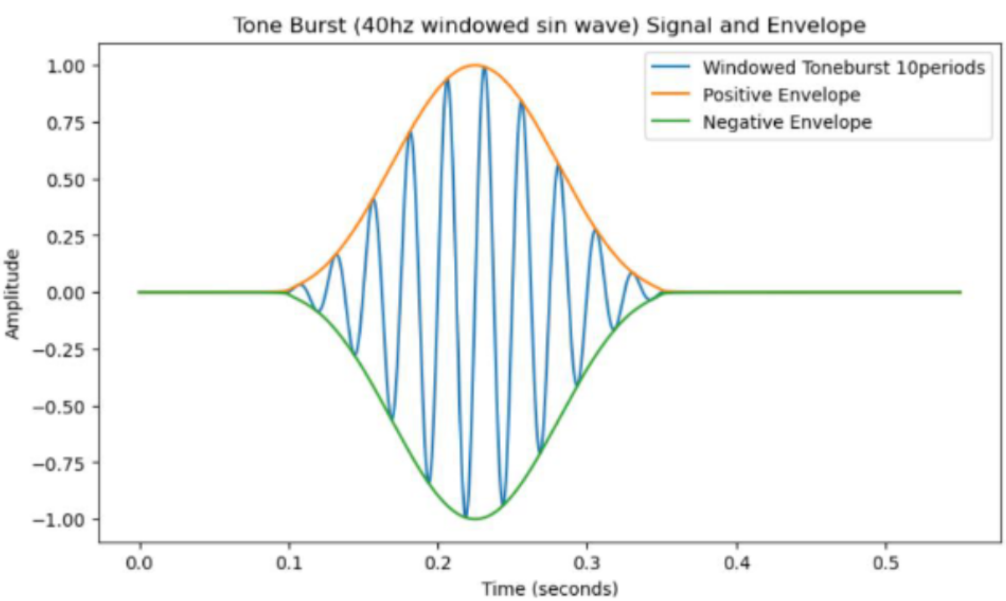
In the above plot, I plotted both the positive and negative envelopes of a windowed (Kaiser) sine wave forming a 10-cycle toneburst. For the rest of the paper, I will only plot the positive envelope, as that is sufficient (so the orange curve above) to see what is happening. The envelope will be useful later when studying the various bass alignments responses to dynamic bass input. If you think this is somewhat like the Energy Time Curve (ETC) frequently seen, you would be correct, both use the Hilbert Transform. The ETC curve is in fact the envelope of the impulse response but plotted differently.



有些人可能會理所當然地選擇「系統 1」作為「穩定」或「衰減」較快，但實際上「系統 2」具有更好的時域性能。直觀地說，缺乏低音的系統的階躍響應會更快地穩定到零，畢竟無限頻寬系統的階躍響應對於所有 t > 0 都是 1。階躍響應「返回零」所需的時間與其低音延伸有關，而不是時域性能。

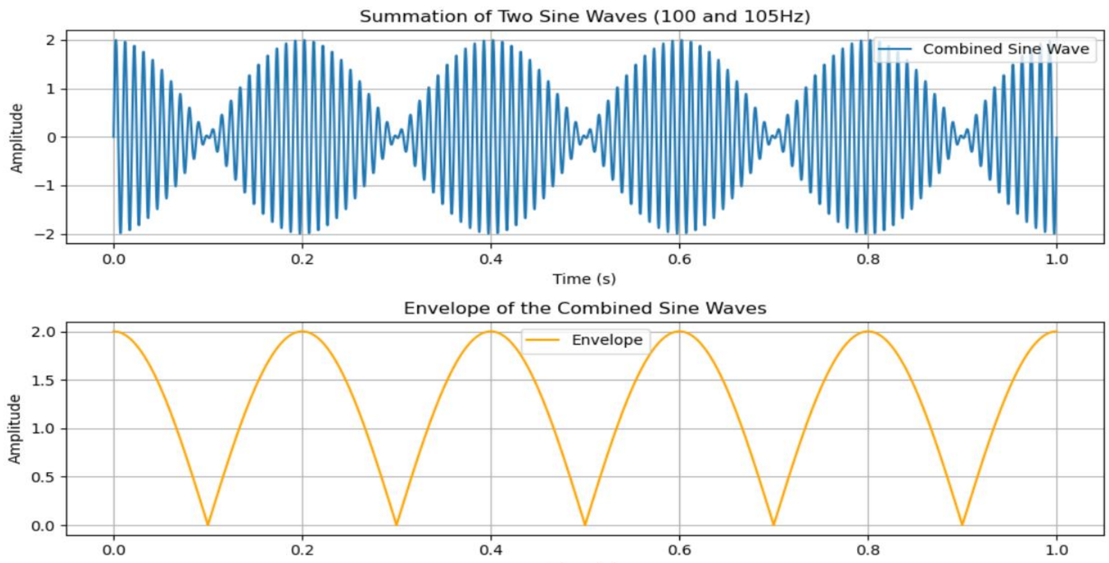
信號包絡簡介（幾乎沒有數學！）

我需要介紹信號包絡，這是一種識別快速變化的複雜信號的瞬時幅度的辦法。一張圖片勝過千言萬語：

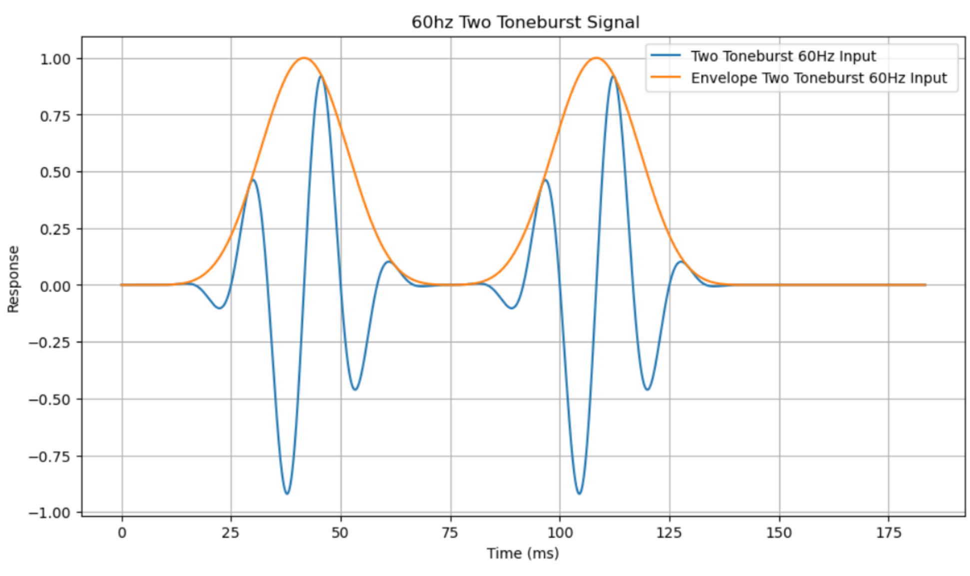


在上面的圖中，我繪製了窗化（Kaiser）正弦波的正包絡線和負包絡線，形成 10 個週期的音爆。在本文的其餘部分，我將只繪製正包絡線，因為這就足夠（因此上面的橙色曲線）可以看到正在發生的事情。在研究各種低音校準對動態低音輸入的響應時，包絡線將在稍後派上用場。如果您認為這有點像經常看到的能量時間曲線 (ETC)，那麼您是正確的，兩者都使用希爾伯特變換。ETC 曲線實際上是脈衝響應的包絡線，但繪製方式不同。

Another example of the envelope of a signal – here is the summation of 2 sine waves (100 and 105Hz) showing the beat-frequency and the envelope shows the beating very clearly.



The toneburst signal (windowed sine wave) is very useful to judging bass time-domain quality, and the envelope is especially useful for judging the amount of "smear" or delay the different filter models have. I am going to use a "two toneburst" signal with 4cycles of each of a given frequency sine wave. Here is the signal and envelope for a 60Hz two toneburst:

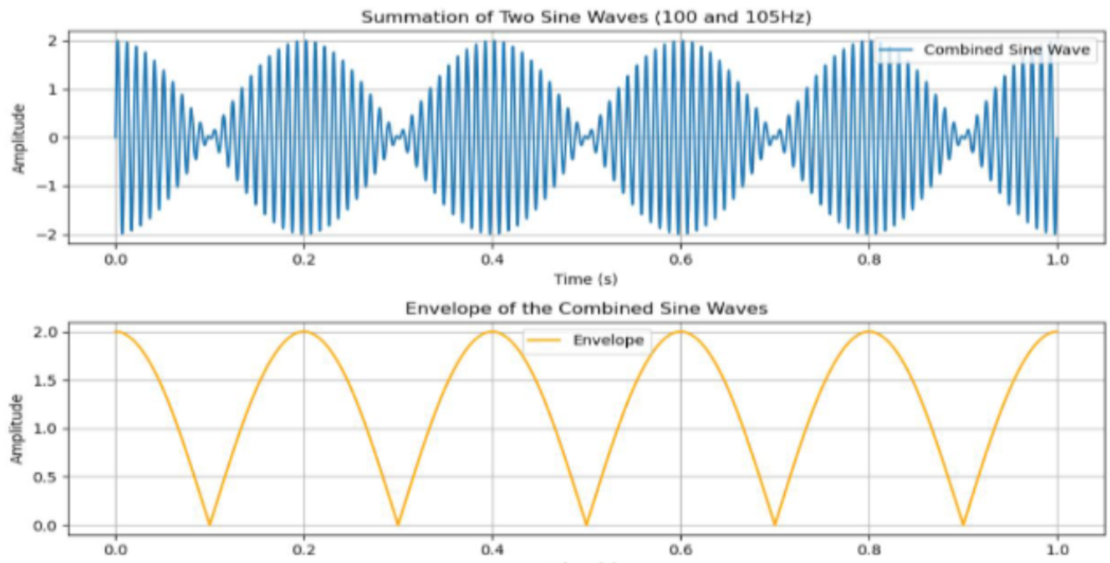


The above envelope shows two clearly separated peaks for the envelope, with the envelope falling to 0 in between.

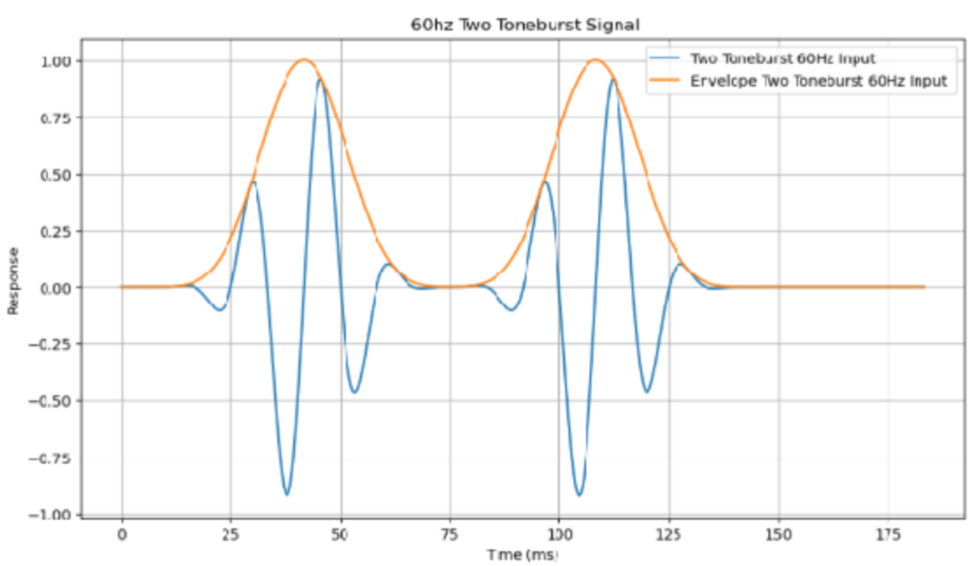
A few selected bass alignments:

It is now time to look at a few different bass alignments. I have experimented with vented and sealed box alignments for years. One system I built was a sixth-order vented box subwoofer (pre-DSP) which produced deep and flat bass in a small cabinet, but despite measuring well it sounded muddy in comparison to a sealed cabinet with the same driver (also driven with op-amp based Linkwitz transform). In all the following examples a low-pass

信號包絡的另一個範例 – 這是 2 個正弦波 (100 和 105Hz) 的總和，顯示拍頻，而包絡非常清楚地顯示拍動。



音爆信號 (窗化的正弦波) 非常適用於判斷低音時域品質，而包絡
特別適用於判斷不同濾波器模型的「拖曳」或延遲量。我將使用
「雙音爆」信號，每個給定頻率正弦波有 4 個週期。以下是 60Hz 雙音爆的信號和包絡：



上述包絡顯示包絡的兩個清楚分開的峰值，而包絡在中間降至 0。

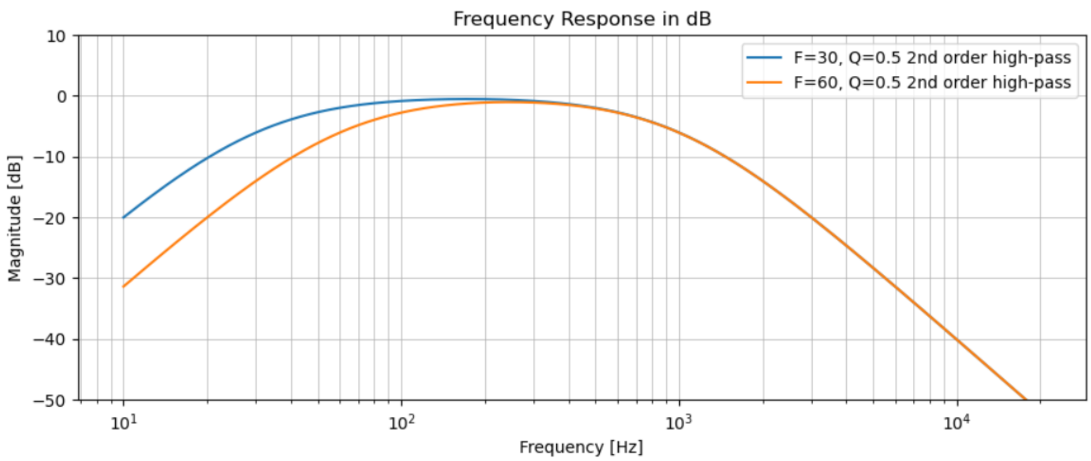
幾個選定的低音排列：

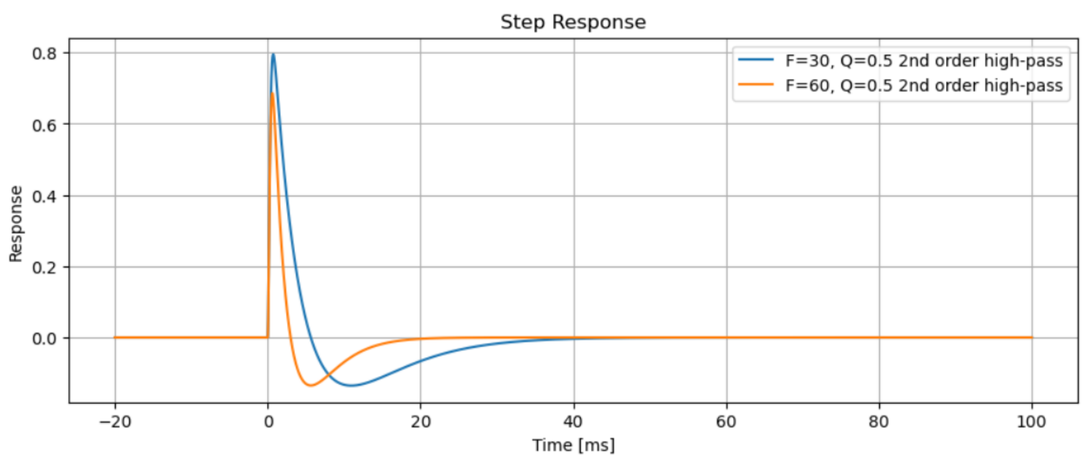
現在是時候來看幾個不同的低音排列。我已經針對通風和密封箱排列進行多年實驗。我建構的一個系統是六階通風箱超低音喇叭 (DSP 前)，它在小型音箱中產生深沉且平坦的低音，但儘管測量良好，與使用相同驅動單體的密封音箱相比，聽起來還是很混濁 (也使用基於運算放大器的 Linkwitz 變換驅動)。在以下所有範例中，低通

Linkwitz-Riley 2nd order filter is applied at 1000hz so the differences in the responses are from the different high-pass filters. Frequency, step and toneburst responses will be shown, along with the envelope for the tonebursts.

Example 1: Two sealed box – same Q, different Fb

Here are frequency and step response graphs for Q=0.5, Fb1=30Hz, Fb2=60Hz:





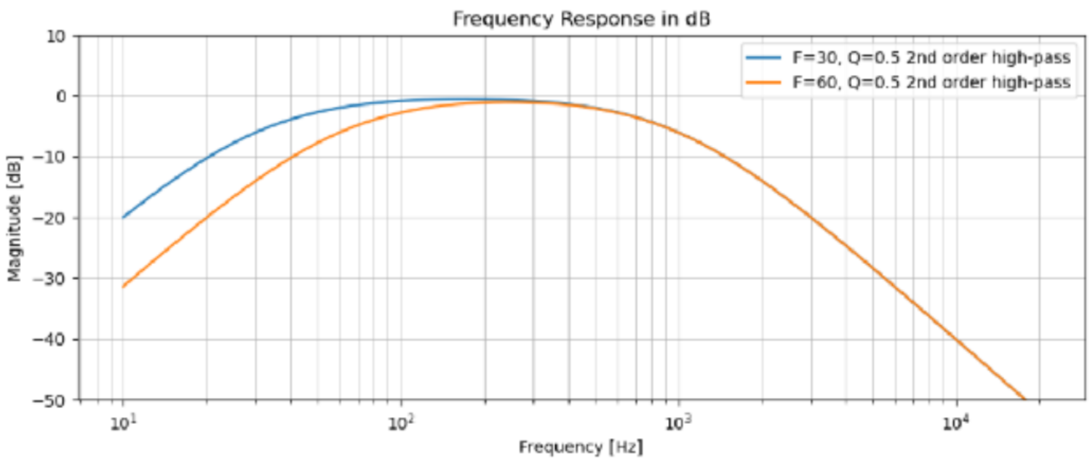
Notice the F=60 appears to "settle faster" than the F=30 case.

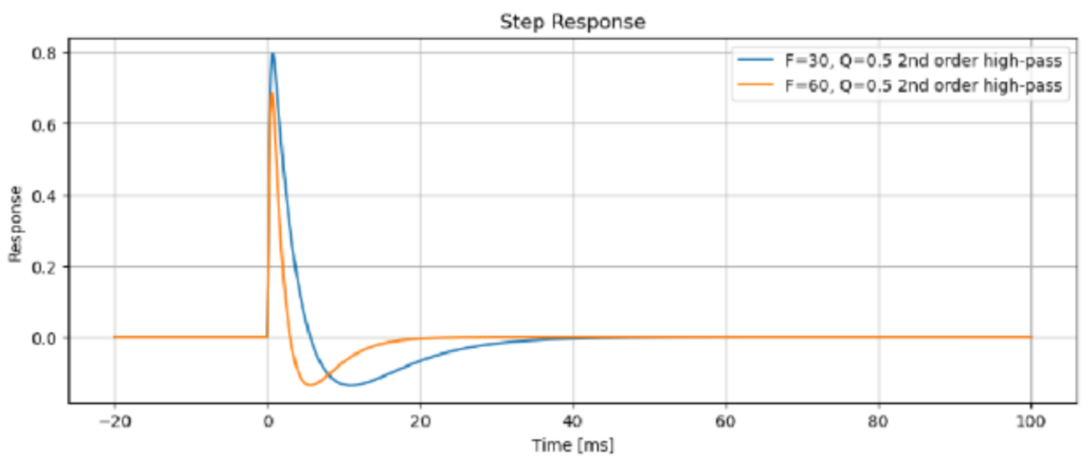
Here are the toneburst responses for 100hz:

Linkwitz-Riley 2order濾波器應用於 1000hz，因此響應的差異來自於不同的高通濾波器。將顯示頻率、階躍和音爆響應，以及音爆的包絡線。

範例 **1**：兩個密閉式音箱 **–** 相同 **Q**，不同的 **Fb**

以下是 Q=0.5、Fb1=30Hz、Fb2=60Hz 的頻率和階躍響應圖形：

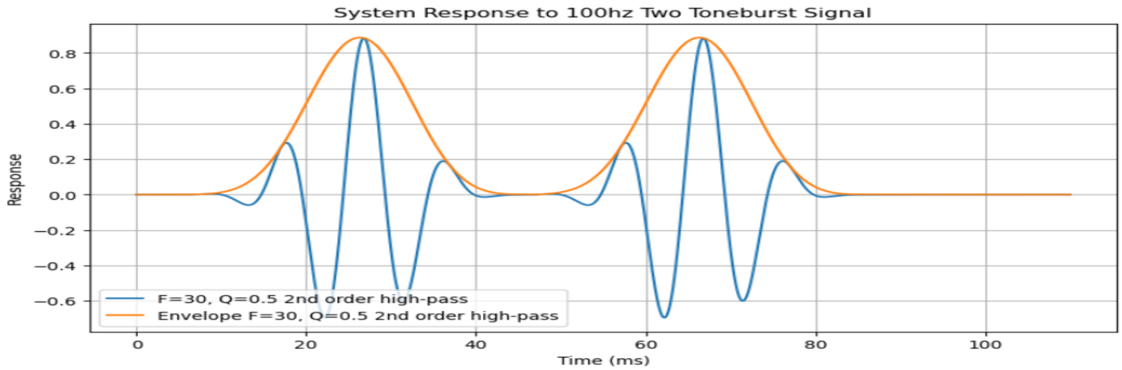


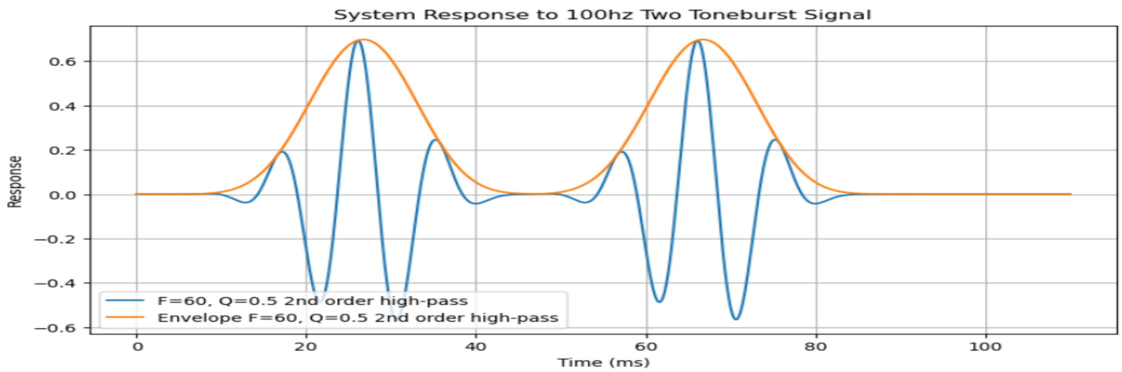


請注意，F=60 看起來比 F=30 的情況「更快穩定下來」。

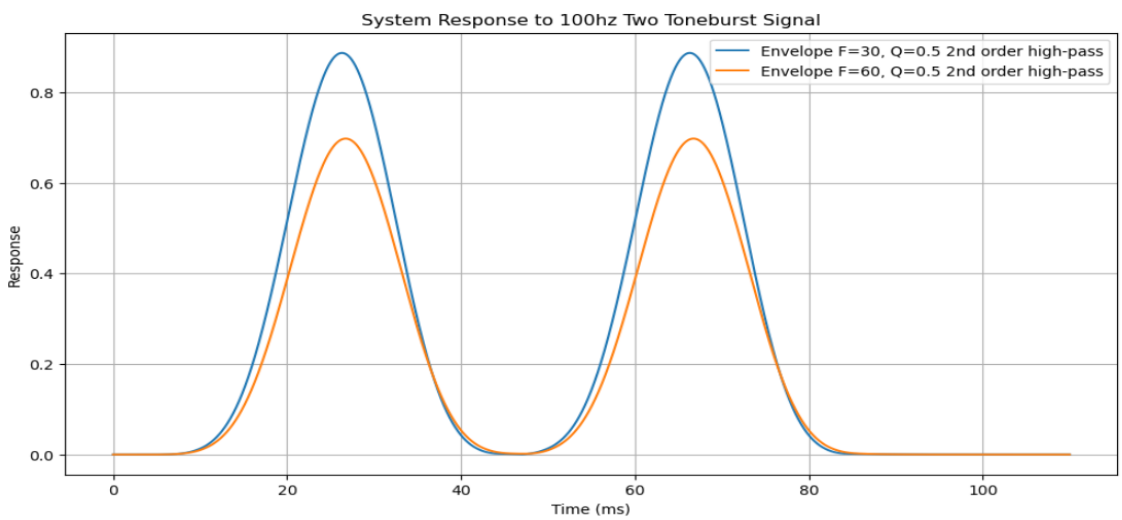
以下是 100hz的音爆響應：

在以下所有範例中，低通 Copyright 2024, Dan Cyr



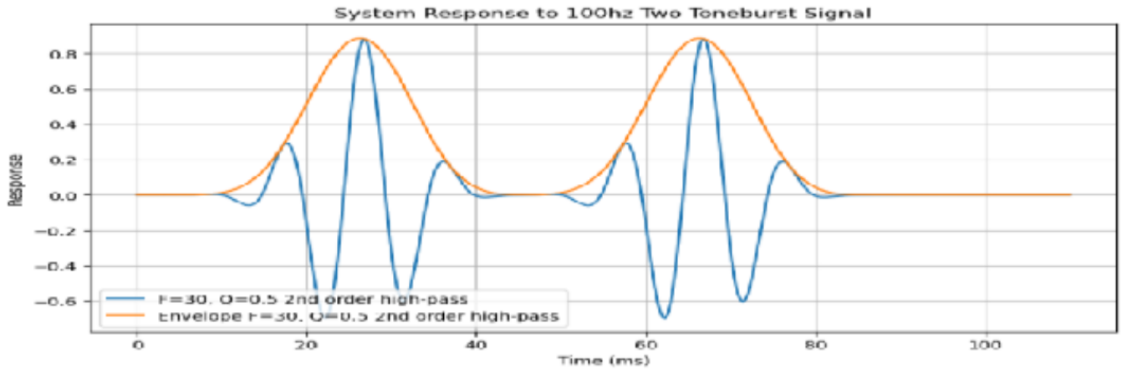


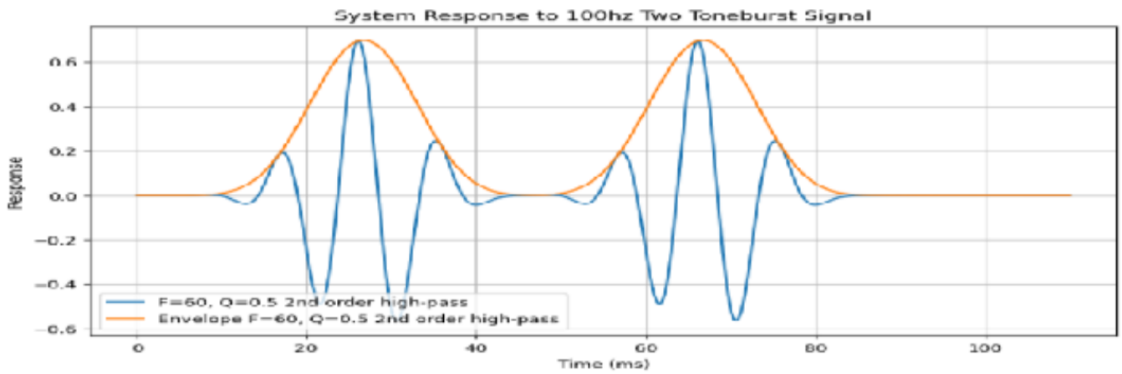
Let's just look at the envelope of each on the same plot:



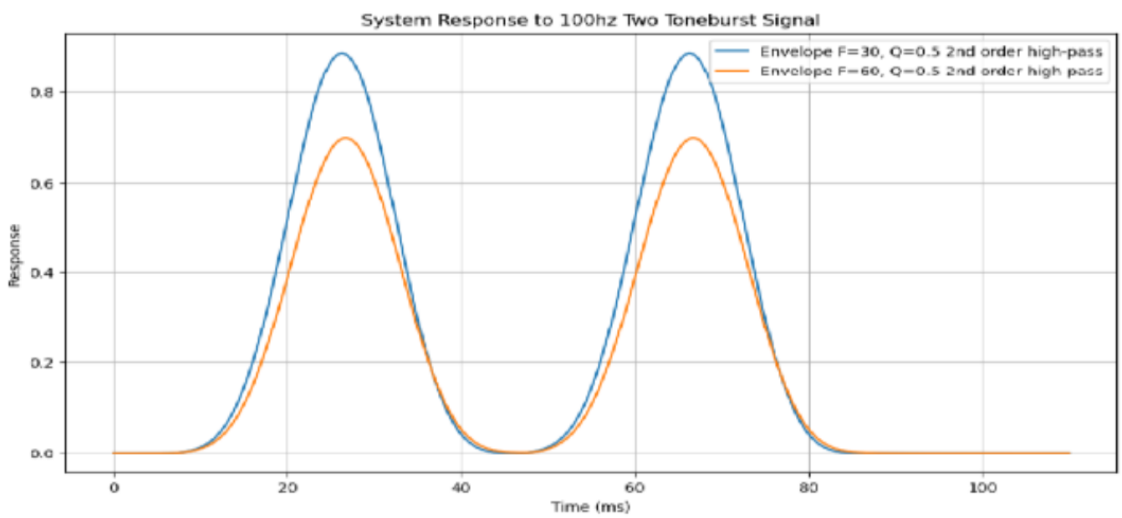
Remember the step-response for the F=30Hz case took a lot longer to "settle" to 0 and looked like it might decay slower. The envelopes of tonebursts show this isn't the case, the f=60 case has less amplitude because it's closer to the filter knee, and Q=0.5 response has a shallow roll off. The f=60 case is slightly delayed compared to the f=30 case.

How about comparing the output envelope to input envelope?



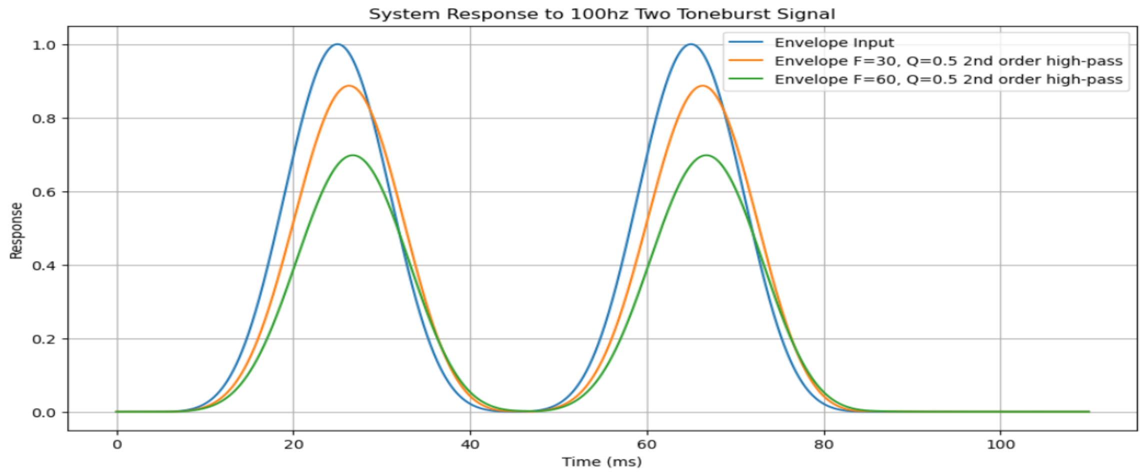


讓我們看看每個信封在同一個圖上的情況：



請記住，F=30Hz 情況的階躍響應花了更長時間才「穩定」到 0，看起來衰減速度可能較慢。音爆的信封顯示情況並非如此，f=60 情況的振幅較小，因為它更接近濾波器膝點，而 Q=0.5 響應的滾降較淺。與 f=30 情況相比，f=60 情況略有延遲。

如何比較輸出信封和輸入信封？

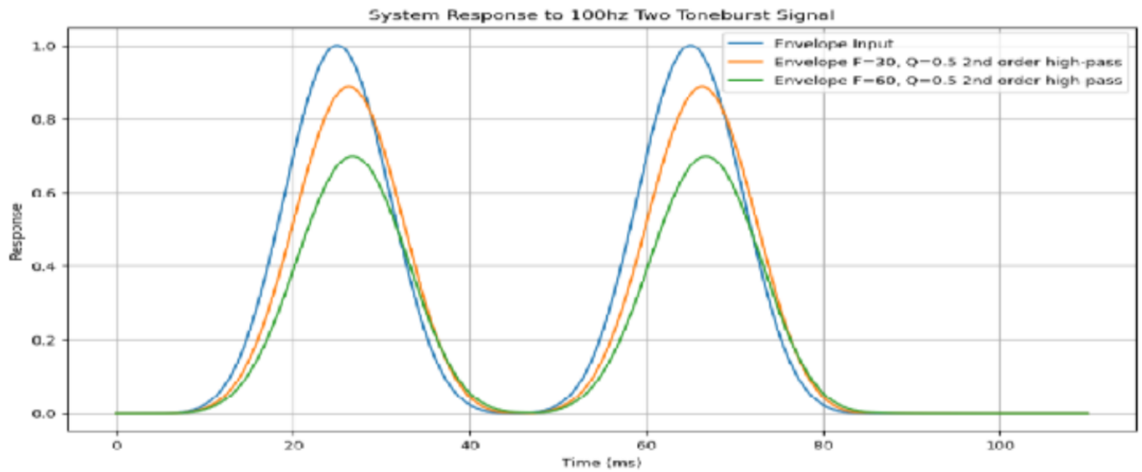


It is clear the output is slightly delayed compared to the input (more so for F=60), but the envelope of the output falls to zero and the peaks are just as separated as the input, and decay is excellent.

The important point here – having a system extend lower in frequency does not hurt the transient response, in fact it helps. The reason of course being the group-delay is lower (at the frequency of the toneburst) for the lower corner frequency filter.

Example 2: Sealed box - Different Fb, Q

Now compare F=30 for Q=0.58 and F=90 Q=1.2 – Sometimes designers with small, sealed boxes will use a high-Q to give it some apparent weight in the bass. The LS3/5 is a classic example. First up is frequency and step response:

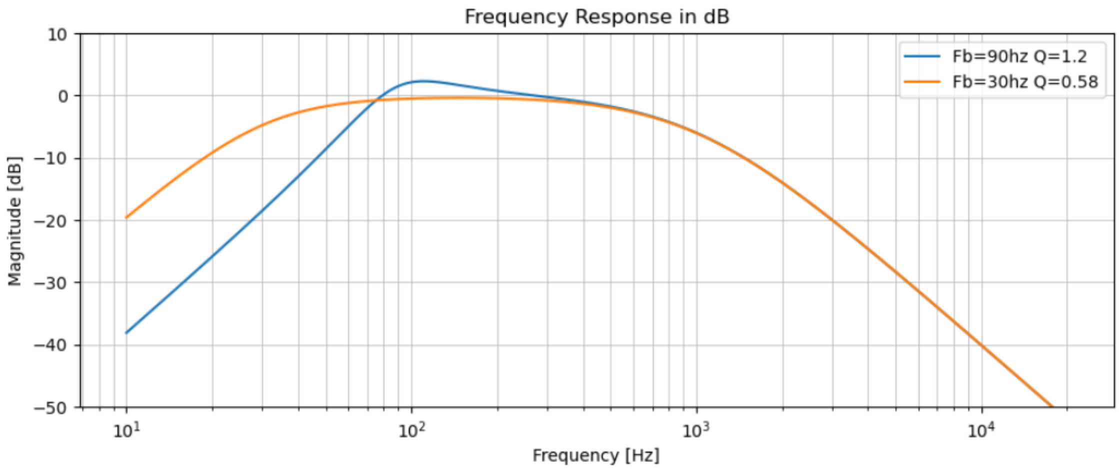


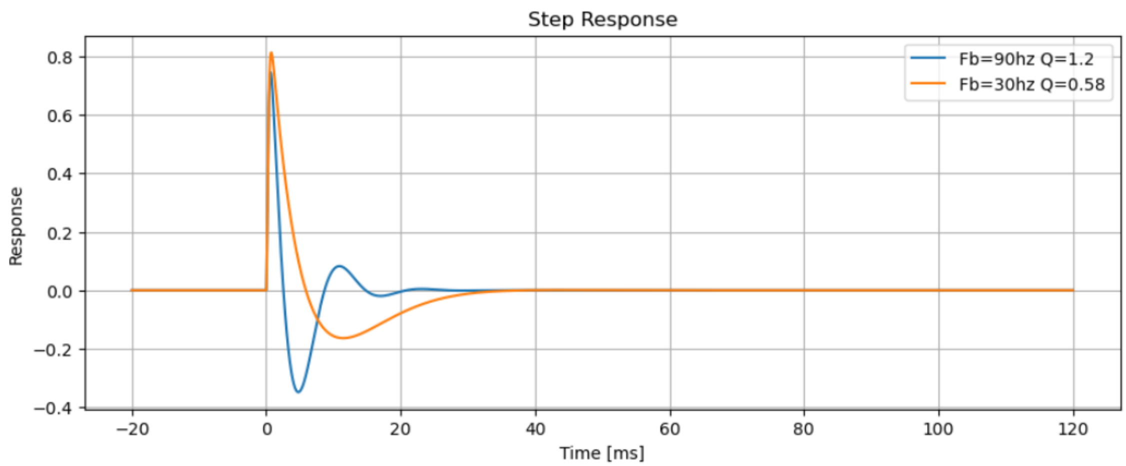
很明顯輸出與輸入相比略有延遲（F=60 更明顯），但輸出的包絡線降至零，峰值與輸入一樣分離，衰減效果極佳。

重點在於 – 系統頻率降低並不會損害瞬態響應，事實上是有幫助的。原因當然是群組延遲較低（在音調爆發的頻率）較低邊緣頻率濾波器。

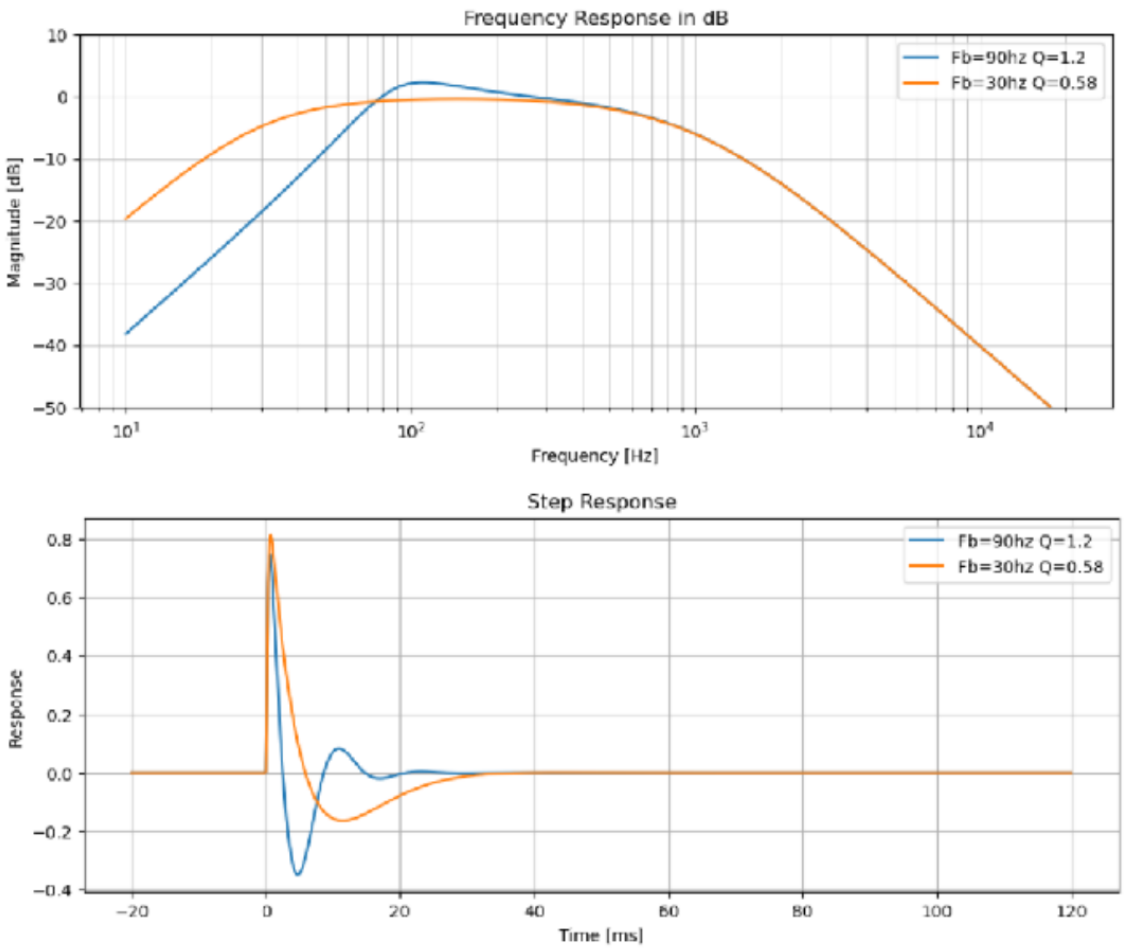
範例 **2**：密閉式音箱 **-** 不同的 **Fb** 、**Q**

現在比較 Q=0.58 的 F=30 和 Q=1.2 的 F=90 – 有時設計師會使用高 Q 值的小型密閉式音箱，讓它在低音中具有一些明顯的重量。LS3/5 就是一個經典的例子。首先是頻率和階躍響應：



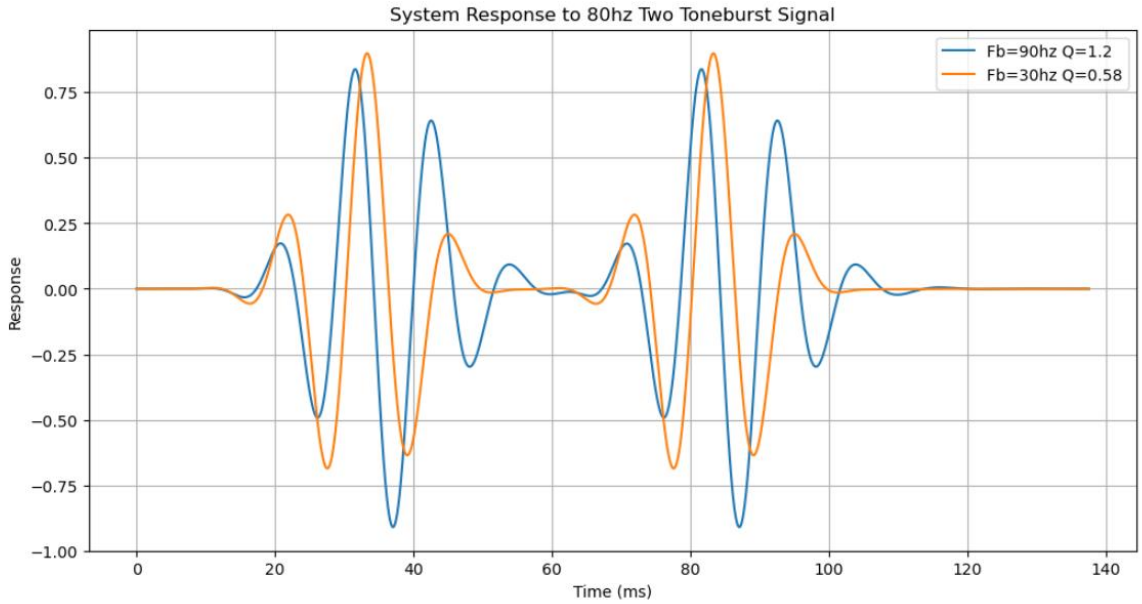


Notice the Q=1.2 case the step response decays quicker than the Q=0.58 and has a small (<3 dB) peak ~100hz.

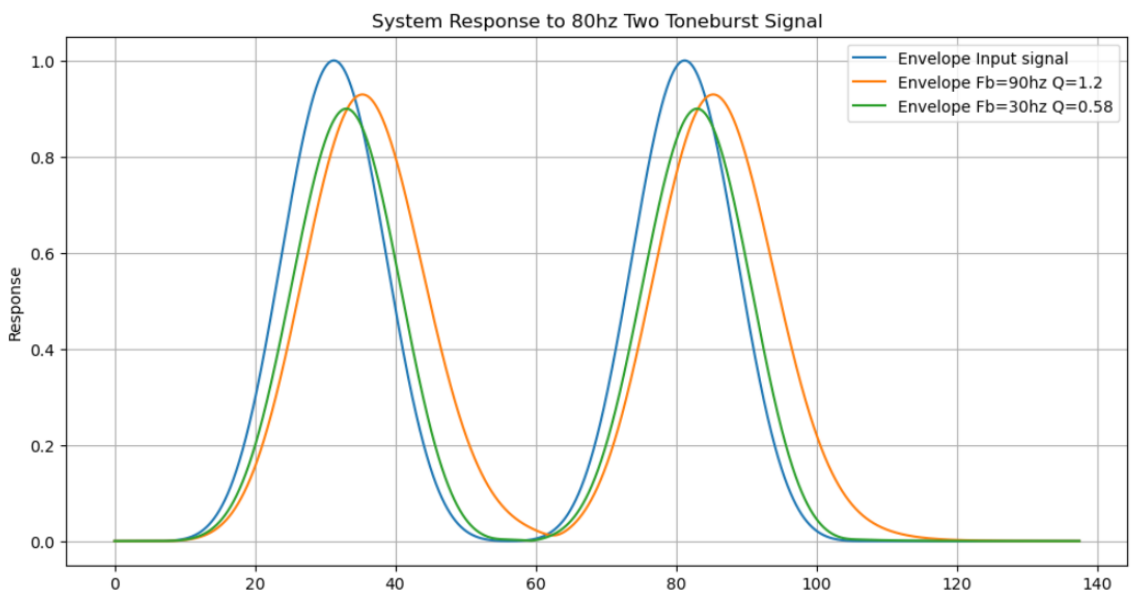


注意 Q=1.2 的情況，階躍響應比 Q=0.58 衰減得更快，並且有一個小的 (<3 dB) 峰值 ~100hz。

Now the toneburst responses:



It looks like the Q=1.2 case is decaying slower – but envelope shows it clearly:

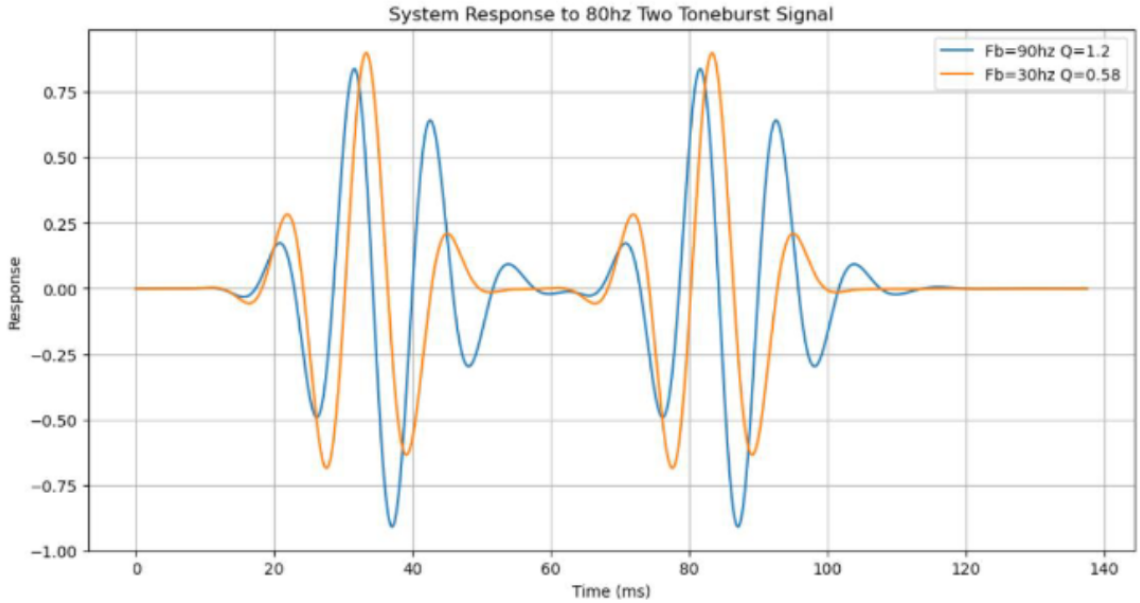


Notice the Q=1.2 case has somewhat smeared the envelope, widening it and reducing the gap between the peaks. While the Q=0.58 looks better, it would require a *MUCH* larger box to achieve or serious amounts of boost & EQ.

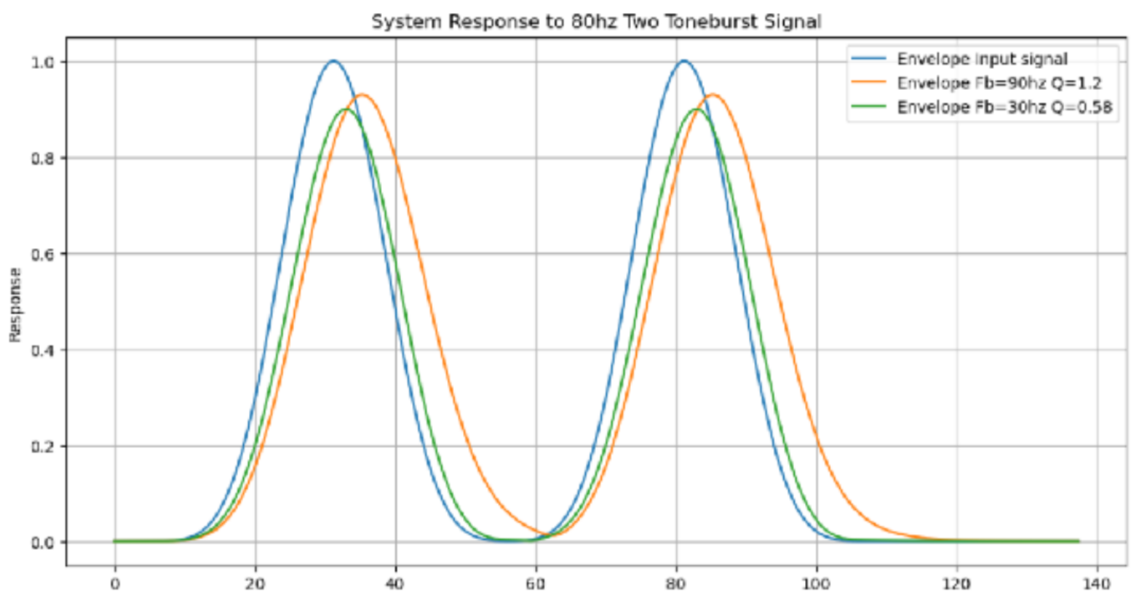
Example 3: Vented, PR or TL box (Butterworth 4^{th} order high pass) and Q=0.58 sealed box

Frequency and step response:

現在的音爆反應：



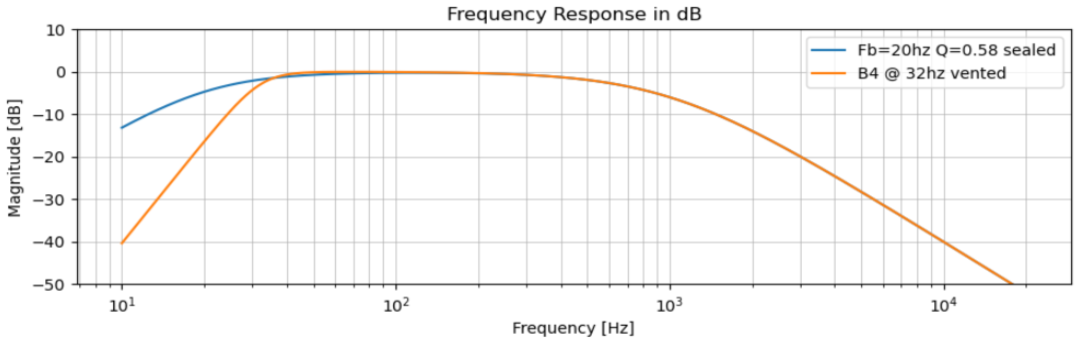
看起來 Q=1.2 的情況衰減較慢 – 但包絡線清楚地顯示：

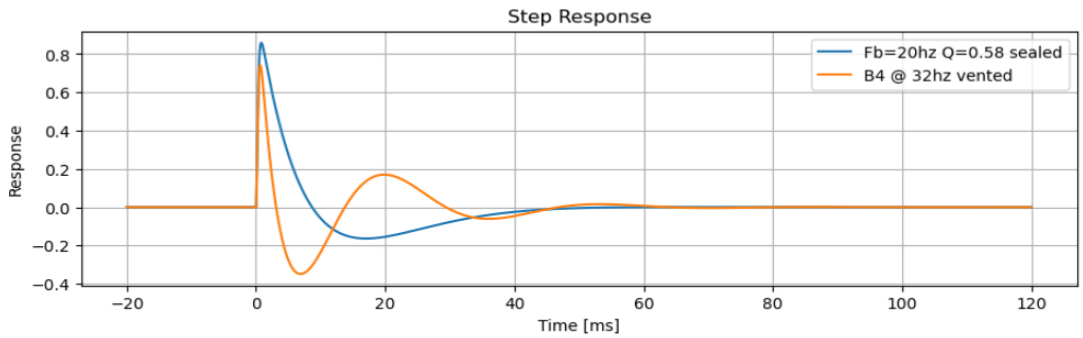


請注意 Q=1.2 的情況已稍微弄髒包絡線，使其變寬並縮小峰值之間的間隙。
雖然 Q=0.58 看起來更好，但它需要一個*大得多的*盒子才能實現或大量提升和 EQ 。

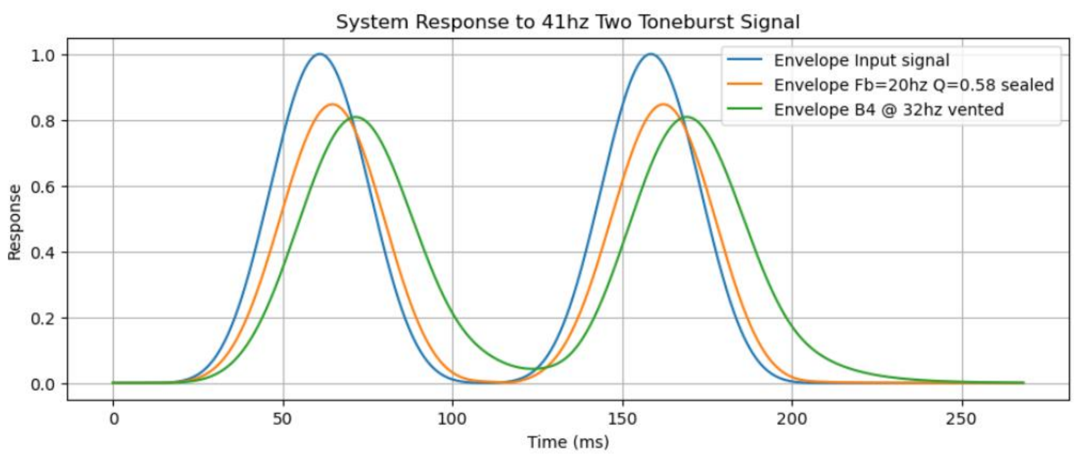
範例 **3**：通風、**PR** 或 **TL** 箱體（巴特沃斯 **4** 階高通）和 **Q=0.58** 密封箱體

頻率和階躍響應：

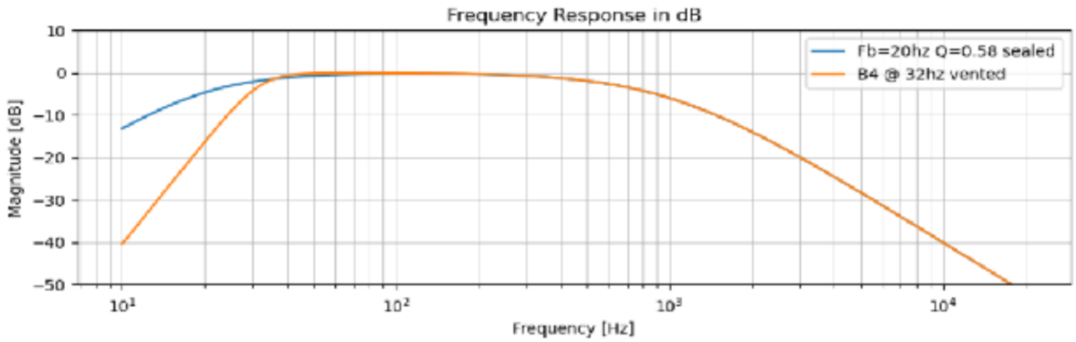


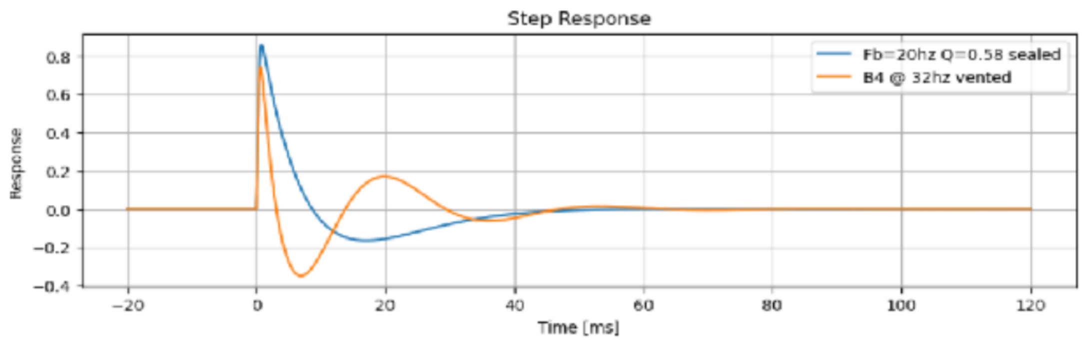


Toneburst Response Envelopes:

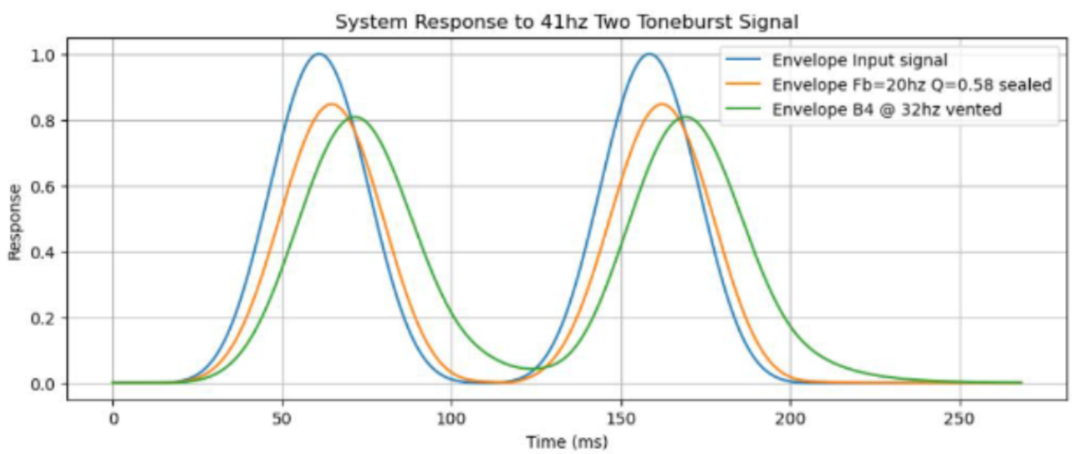


Notice the envelope does not fall to 0, showing the decay is inferior for the B4 alignment, this shows the superior performance of the lower frequency and lower order alignment.



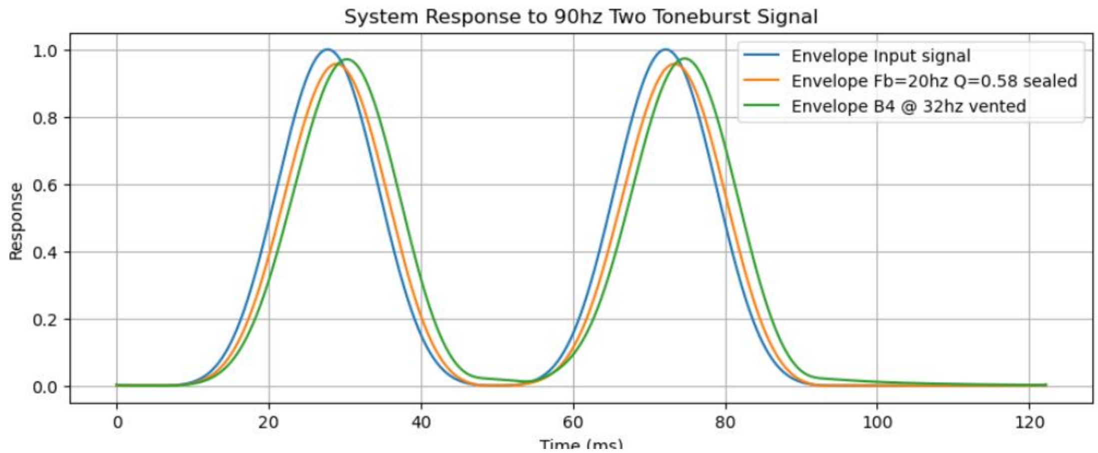


音爆響應包絡：



注意信封不會降至 0，顯示 B4 對齊的衰減較差，這顯示較低頻率和較低階對齊的優異性能。

I have been choosing the frequency of the toneburst so that differences are easier to see – but even at nearly triple the tuning frequency of the vented box, the sealed box has slightly better transient response:

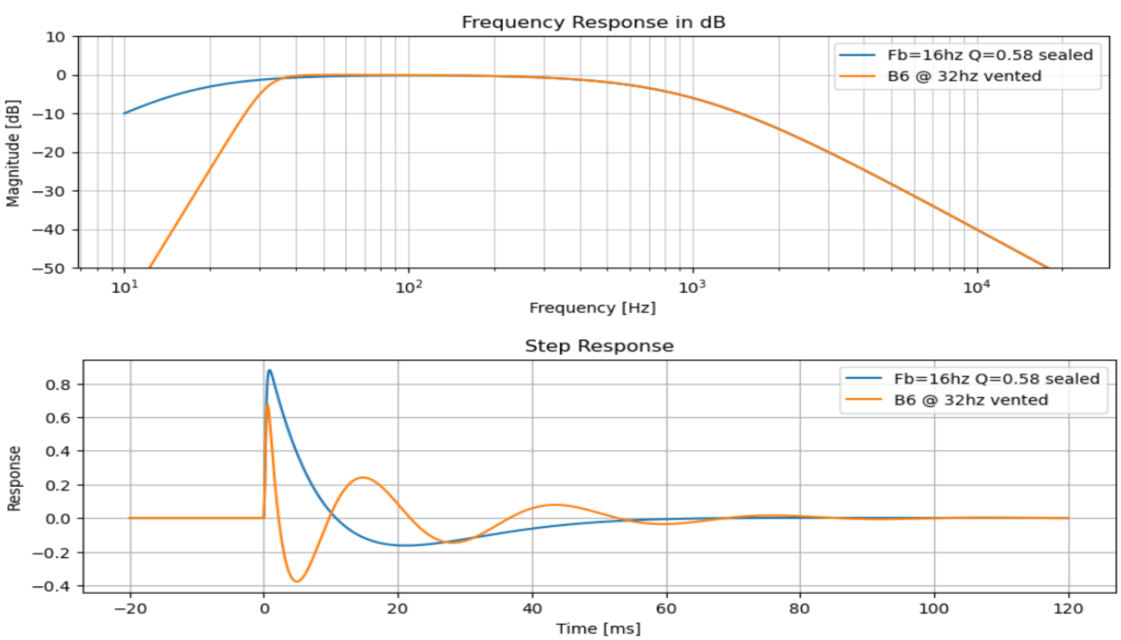


That is not a huge difference, but it's still there, very far away from the tuning frequency, note the changing horizontal time scale with the toneburst frequency change.

Final Example: Vented 6th order Butterworth vs large Q=0.58 equalized sealed box.

I mentioned having built a vented 6th order woofer box before and now this is a worst-case scenario: a higher-order resonant boosted bass alignment vs what I think is the optimal sealed box.

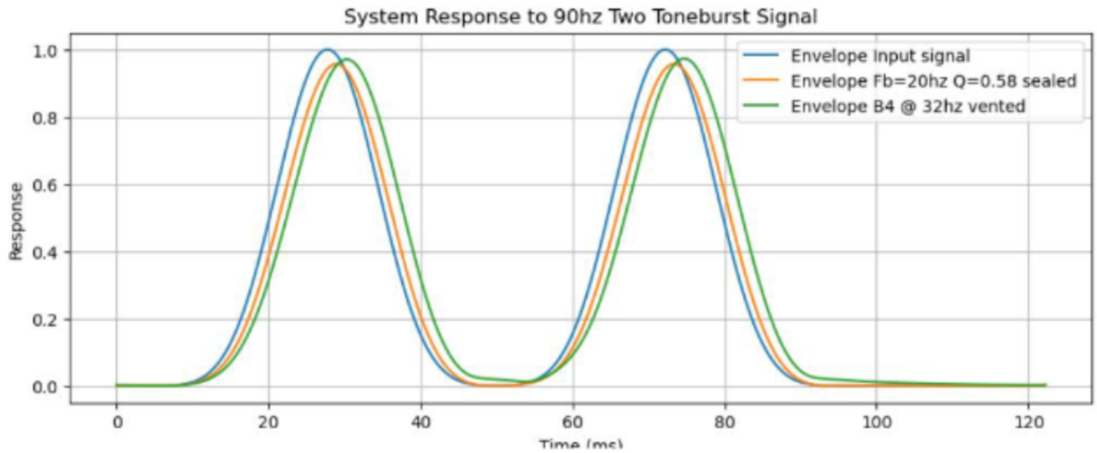
Frequency& Step response:



This is pretty poor performance for the B6 compared to Q=0.58.

Toneburst response:

我選擇音爆的頻率，以便更容易看到差異 – 但即使在通風箱的調音頻率的三倍下，密封箱的瞬態響應略好：

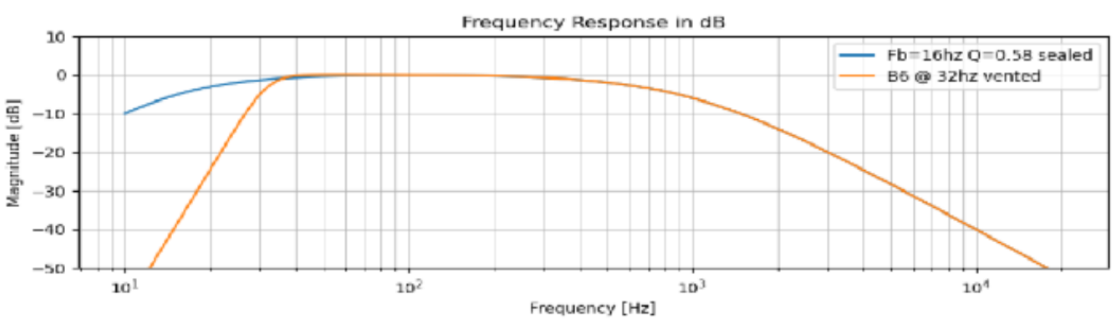


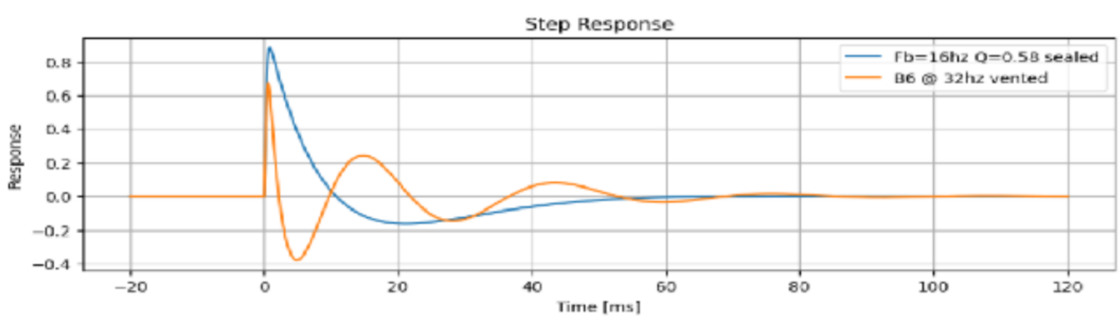
這不是很大的差異，但它仍然存在，遠遠低於調音頻率，請注意音爆頻率變化時水平時間刻度的變化。

最後一個範例：通風 **6** 階巴特沃斯濾波器與 **Q=0.58** 等化的密封箱。

我提到之前曾建構過一個通風 6 階低音揚聲器箱，現在這是最壞的情況：高階共振增強的低音校準與我認為最佳的密封箱。

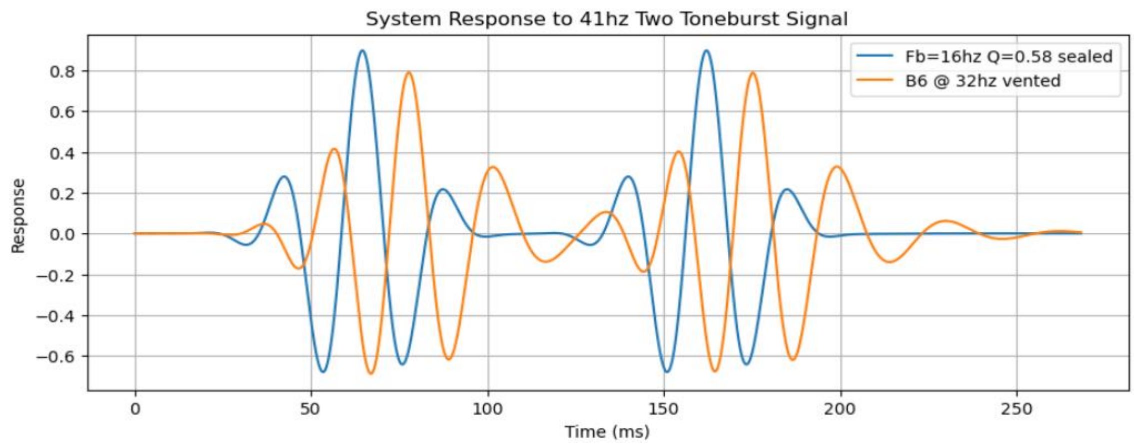
頻率和階躍響應：



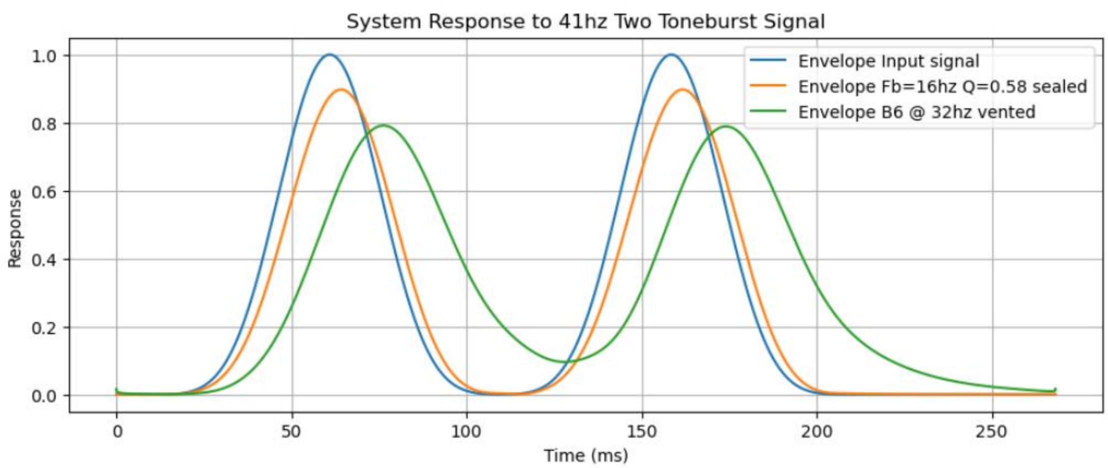


與 Q=0.58 相比，這是 B6 非常差的性能。

音爆反應：



Envelope:



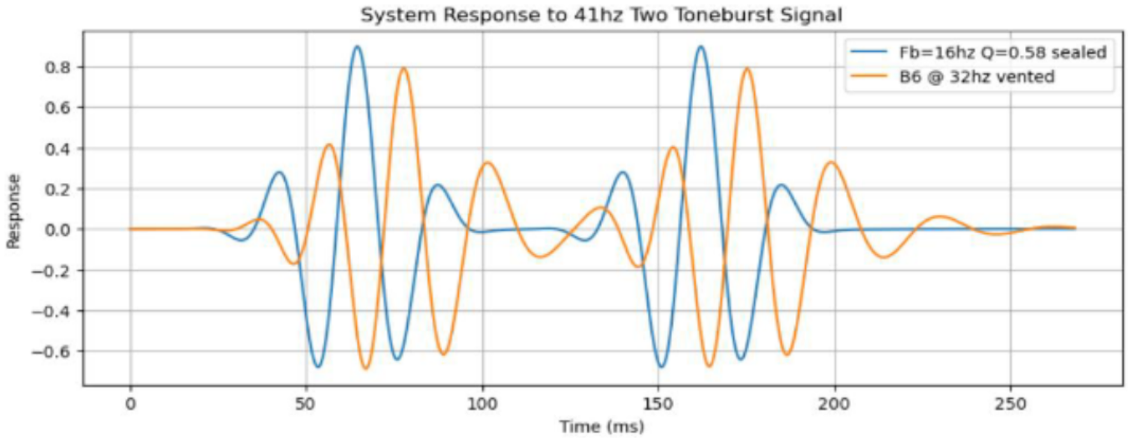
The B6 system has significant delay as well as the envelope never decaying near zero between the two tonebursts. I

Final thoughts:

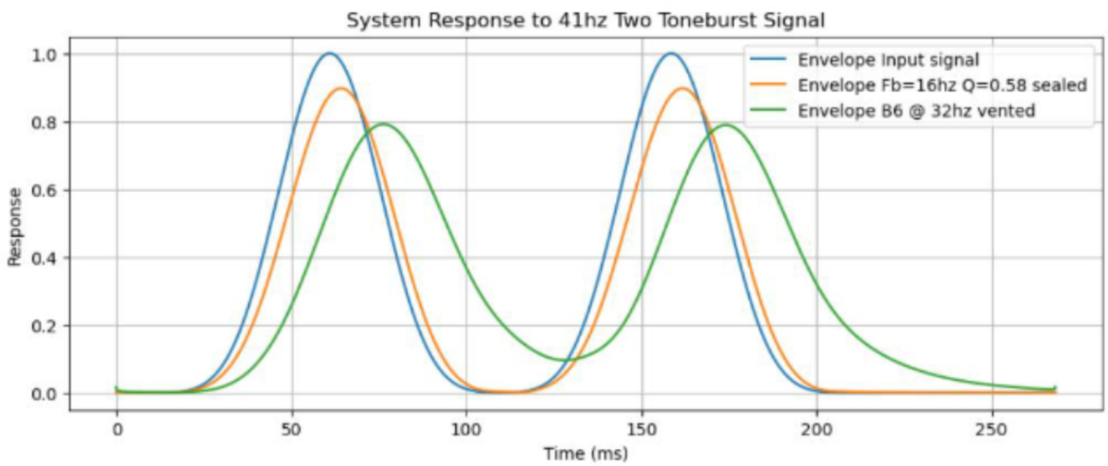
The higher the order of the high-pass response, the worse the transient response, within limits. For example, the Q=1.2 sealed box did less smearing than B4 or B6 alignments. There is never a case that extending the bass response "hurts" performance, however room gain should be measured and designed for, it's possible a higher Fc and lower Q will be needed. When placed in a room, room modes sometimes take time to build up resonance, and having a better decay time is useful to reduce their effect.

If you are the designer who wants to use a full-range driver (to avoid having crossover cause phase rotation) in a transmission line, perhaps realizing your bass alignment is doing far more damage to transient response than a proper crossover would be in order. Speaker design is balancing tradeoffs – and these results shouldn't be used in a religious war about sealed box vs. vented. Good sounding designs exist for each, and ultimately the audibility of the superior transient response of sealed box is still an open question.

I think for DIY having plentiful class D power and DSP it is very hard to argue against heavy EQ of a sealed box if transient performance is a goal. I would be less inclined to use in a PA setting where cooling of voice-coils and maximal output is desired, let alone the better distortion figure near box tuning (if designed correctly) a vented box can have.



信封：



B6 系統有顯著的延遲，而且在兩個音爆之間，包絡線從未衰減至接近零。I

最後的想法：

高通響應的階數越高，瞬態響應越差，但也有限度。例如，Q=1.2 密閉箱比 B4 或 B6 對齊的音染更少。擴展低音

響應「損害」性能的情況從不存在，但應測量並設計房間增益，可能需要更高的 Fc
和更低的 Q。放在房間時，房間模式有時需要時間才能建立共振，而更好的衰減時間有助於減少其影響。

如果您是設計師，想在傳輸線中使用全音域驅動器（以避免分頻器造成相位旋轉），那麼您可能會意識到，與適當的分頻器相比，您的低音對齊對瞬態響應造成的損害更大。揚聲器設計是權衡取捨——這些結果不應被用於密閉箱與倒相箱的宗教戰爭中。每種都有好聽的設計，而且最終密閉箱優異瞬態響應的可聽性仍然是一個懸而未決的問題。

對於 DIY 來說，如果瞬態效能是目標，那麼很難反駁對密閉式音箱進行大量 EQ 調整，因為它具有充足的 D 類功率和 DSP 。在需要冷卻音圈和最大輸出功率的 PA 環境中，我比較不傾向使用它，更不用說經過正確設計後，通風式音箱在箱體調音附近可以具有更好的失真數據。